Maximally symmetric spaces in Brans-Dicke theory and the cosmological constant

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Abstract

In this note we discuss a toy model in which the value of the effective cosmological constant is associated with a symmetry properties of the vacuum background solution and defined by the model parameter associated with a scalar field non-minimally coupled to gravity. Although the parameters in the model seem to have unrealistic values to account for the present value of the cosmological constant, the model demonstrates a possible alternative way it can arise.

During the last years the problem of cosmological constant, its origin and small value attracts much attention. It is possible that the vacuum energy is indeed a constant, and there are a lot of attempts to explain the existence of such constant vacuum energy – see, for example, reviews [1, 2] and references therein. One of the most interesting questions is that about its extremely small value. Indeed, naive consideration show that the vacuum energy value should be many orders of magnitude larger. In this note we discuss a toy model, which does not solve this problem, but proposes a possible way to change the origin of the cosmological constant.

First, let us consider the action

$$S = \int d^4x\sqrt{-g}\left[\phi R - \omega\frac{\partial^\mu\phi\partial_\mu\phi}{\phi} - V(\phi)\right], \tag{1}$$

where R is the four-dimensional curvature, ϕ is the Brans-Dicke field, ω is the dimensionless Brans-Dicke parameter. The equation of motion for the Brans-Dicke field ϕ, coming from this action, has the form

$$(2\omega+3)\,\nabla^\mu\nabla_\mu\phi + 2V(\phi) - \phi\frac{dV(\phi)}{d\phi} = 0, \tag{2}$$

where ∇_μ is the covariant derivative with respect to the metric. For the case $V(\phi) = 0$ we get the well known equation of the original Brans-Dicke theory [3]

$$(2\omega+3)\,\nabla^\mu\nabla_\mu\phi = 0. \tag{3}$$

In this case the static background solution of Einstein and scalar field equations has the form

$$g_{\mu\nu} = \eta_{\mu\nu}, \tag{4}$$

$$\phi = const, \tag{5}$$

where $\eta_{\mu\nu}$ is the Minkowski metric.

Now let us consider another possibility. Namely, let us take

$$V(\phi) = \gamma\phi^2, \tag{6}$$

where γ is a dimensionless constant (cosmology of Brans-Dicke theory with such form of the potential was discussed in [4]). The scalar field equation for the choice (6) again takes a simple

form (3). It is evident that there exists a static solution of equation (3) $\phi = const$. As for the metric, corresponding background solutions take the form of dS_4 for $\gamma > 0$ and AdS_4 for $\gamma < 0$, which can be easily seen from the Einstein equations [3]:

$$\phi\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) = -\frac{1}{2}g_{\mu\nu}\gamma\phi^2 + \tag{7}$$

$$+\frac{\omega}{\phi}\left(\nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2}g_{\mu\nu}\nabla^\rho\phi\nabla_\rho\phi\right) + \nabla_\mu\nabla_\nu\phi - g_{\mu\nu}\nabla^\rho\nabla_\rho\phi.$$

Thus, regardless of the sign of γ, there exist solutions which describe maximally symmetric spaces (which are Minkowski for $\gamma = 0$, dS_4 or AdS_4 for $\gamma \neq 0$). This result is well known. Indeed, passing from the Jordan frame to the Einstein frame with the help of conformal transformations results in reducing the scalar field potential to the constant potential $V^*(\phi) \sim \gamma$. An important point is that though solutions in different frames are equivalent, the corresponding theories in different frames are not equivalent, because if in the Brans-Dicke theory we had the minimal coupling of gravity to any additional matter, then after the conformal transformation the scalar field would enter the term describing the interaction with matter (the so-called conformal ambiguity [5]). Thus, if we suppose that the physical metric is that of the Jordan frame, it is more convenient to use original action of the form (1).

The situation radically changes if $V \neq \gamma\phi^2$, for example, if $V = \gamma\phi^2 + \Lambda$, where Λ is a constant. One can check that in this case $\phi \neq const$ and solution for the metric does not correspond to a maximally symmetric space. Thus, a maximally symmetric spaces can be realized only if $\Lambda = 0$ (we note, that in the absence of the Brans-Dicke scalar field, i.e. in the standard case, the maximally symmetric spaces exist for any value of Λ). But it is not evident why it should be so. Nevertheless, one can recall that there exists the well-known mechanism based on introducing the 3-form gauge field into the theory [6, 7, 8, 9], which turns the cosmological constant into an integration constant.

To this end let us consider the action of the form

$$S = \int d^4x\sqrt{-g}\left[\phi R - \omega\frac{\partial^\mu\phi\partial_\mu\phi}{\phi} - \gamma\phi^2 - \bar{\Lambda} - \frac{1}{48}F_{\mu\nu\rho\sigma}F^{\mu\nu\rho\sigma} + L_{matter}\right], \tag{8}$$

where $\bar{\Lambda} > 0$ is the "bare" energy density of the vacuum and supposed to include, for example, the contribution of quantum fluctuations, thus its value can be large, L_{matter} stands for the Lagrangian of other matter (we would like to mention again that the metric in this frame is supposed to be the physical metric). The 3-form gauge field arises in supergravity theories [6], but in the toy model under consideration it is introduced without reference to any particular theory.

The solution of equations of motion for the field strength of the 3-form gauge field is

$$F^{\mu\nu\rho\sigma} = \frac{c\epsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}}, \tag{9}$$

where c is a constant, and the contribution of this 3-form field to the action reduces to

$$-\frac{1}{48}F_{\mu\nu\rho\sigma}F^{\mu\nu\rho\sigma} \to +\frac{c^2}{2}. \tag{10}$$

The constant c in (9) is not fixed by the equations of motion for the 3-form field. Solution (9) is valid for any metric, and once the constant c is fixed, it remains unchanged. The 3-form field appears to be non-dynamical.

The equation of motion for the Brans-Dicke field ϕ (2) now takes the form

$$(2\omega + 3)\, \nabla^\mu \nabla_\mu \phi + 2\bar{\Lambda} - c^2 = 8\pi T. \tag{11}$$

In principle the constant c can be arbitrary, leading to an arbitrary value of the effective energy density $\bar{\Lambda} - c^2/2 + \gamma\phi^2$. There are several methods to fix its value in the theory without non-minimally coupled scalar field, see, for example, [1], resulting in the vanishing of the effective cosmological constant. Here we would like to propose another possible method. Indeed, since the cosmological constant is treated as a vacuum energy density, let us consider a vacuum solutions for system (8) (i.e. in the case $T_{\mu\nu} = 0$). There are different solutions depending on the value of c, mainly with time-dependent ϕ and, consequently, time-dependent metric (we are interested in isotropic solutions). But there is also a class of solutions, which possesses an additional property – a symmetry. Thus, we apply the symmetry criterion for choosing the value of constant c: we suppose that the (global) vacuum background solution is maximally symmetric. We note that we do not propose any particular mechanism leading to such choice of constant c. Moreover it is not easy to justify this ad hoc hypothesis, but in this connection one can recall the ideas that the Universe could have global topology and structure, which were discussed, for example, in [10, 11]. Nevertheless, our hypothesis is nothing but an assumption, which can not be consistently justified within the framework of classical theory, and one can only suppose that some more general theory could deal with symmetry properties of a space-time.

The proposed assumption uniquely defines the value of c (since the scalar field ϕ should be constant for the maximally symmetric solutions):

$$c^2 = 2\bar{\Lambda}. \tag{12}$$

Thus, the 3-form field totally compensates the contribution of $\bar{\Lambda}$ to the energy density of the vacuum. It is evident that the background value of the scalar field should be identified with the Planck mass, $\phi = M_{Pl}^2 = const$. If $\gamma > 0$, we would get dS_4 background solution, i.e. the case of positive cosmological constant. In the limit $x^0 \to \infty$ the ordinary and dark matter average densities tend to zero, $\phi \to M_{Pl}^2$, solutions for the metric and the scalar field tend to this vacuum solution. Of course the existence of matter, described by $T_{\mu\nu}$, breaks the symmetry of the vacuum background solution.

Now the contribution to the effective cosmological constant is defined by the term $\gamma\phi^2$ and

$$\rho_{vac} \sim \gamma M_{Pl}^4.$$

Of course, the value of γ should be extremely small to reproduce the present value of the cosmological constant. Such a small value seems to be unrealistic. Moreover, the problem of the small value of parameter γ is not better than the original cosmological constant problem. There are other disadvantages of this scenario. For example, the mechanism discussed above works only for the case $\bar{\Lambda} = const$. We also do not discuss in this note the mechanism which sets the vacuum expectation value of the Brans-Dicke field. It is possible that there could appear a mass term $\sim M_{Pl}^2$ for the field $\varphi = \phi - M_{Pl}^2$, in this case the backreaction of the Brans-Dicke field on the metric fluctuations appears to be totally negligible. But one should take into account that the simple model presented above is nothing but a toy model, which nevertheless can be interesting from the theoretical point of view, because it demonstrates that there could be a connection between a value of the effective cosmological constant and a symmetry properties of the space-time, leading to a possibility that the effective cosmological constant is defined by some theory non-minimally coupled to gravity, whereas the proper energy density of the vacuum $\bar{\Lambda}$ does not contribute to the effective cosmological constant at all.

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